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## REPORT

BALLISTIC DATA FOR FLAT FIRE

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## Abstract

The purpose of this paper is to exhibit a new form of the Hitchcock-Kent Siacci treatment of flat fire ballistics. This form has proved convenient in certain aircraft-fire computation.

The principal feature of the new form is that time of flight and drop are tabulated directly as functions of distance along the line of fire. ("Ground coordinates" are used). Two "Siacci trajectories" and interpolation suffice to give, with fair accuracy, ballistic data for forward fire from aircraft for all airspeeds and densities.

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## Introduction:

In issuing ballistic data for forward fire from pursuit aircraft certain condensed forms appeared desirable. The tabular data, as furnished on this project, consisted of $\sigma t$, $\sigma^2 y$, listed against $\sigma x$, where $\sigma$ is the ratio of density at the muzzle to standard density, t, y, and x are respectively time of flight, drop, and distance along the line of fire. Certain advantages of this form are set forth in the following.

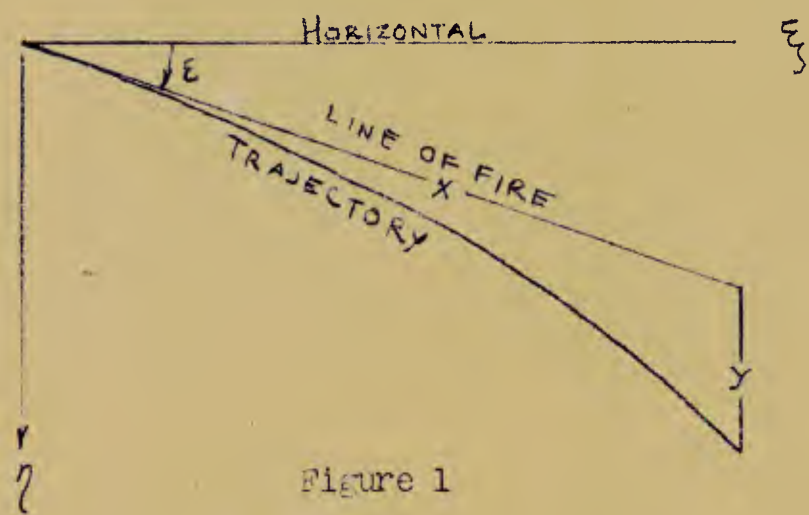



Figure 1

## 1. Modified Siacci Equations.

Referred to mutually perpendicular axes, $\xi\eta$, in the plane of the trajectory with origin at the muzzle of the gun and the y-axis pointing vertically downward (see Figure 1), the (particle) equations of motion may be expressed as

$$\ddot{\xi} = - E\dot{\xi} \tag{1.1}$$
$$\ddot{\eta} = - E\dot{\eta} + g$$

where,

$$E = \frac{\rho(\eta)}{\rho_0} G(v, a, \rho_0) \frac{1}{c} \tag{1.2}$$

and v denotes the velocity of the projectile at time t,
c denotes the ballistic coefficient of the projectile,
$\rho(\eta)$ represents air density at distance $\eta$ from the horizontal axis,
$\rho_0$ represents standard density at the muzzle,

a is the velocity of sound in air at temperature T (absolute temperature) and has the form

$$a = a_s \sqrt{T/T_s}$$

where $a_s$ = velocity of sound in air at standard temperature at sea level,
$T_s$, ($T_s$ = 518.6° F on the absolute scale which corresponds to 59° F)

$G(v, a, \rho)$ is the resistance function which is defined as $\rho v K_D(v/a)$, where $K_D$ is the drag function of the projectile and is experimentally determined.

In this form, the equations present two computational difficulties:

1. The resistance function is tabulated from experiments for standard temperatures and standard density at sea level, $\rho_s$, i.e.

$$G(v) = G(v, a_s, \rho_s).$$

Therefore, $G(v, a, \rho_o)$ must be related to $G(v, a_s, \rho_s)$.

2. In aircraft gunfire the altitude of the muzzle changes. In the form in which E is taken above, changing the altitude of the muzzle requires recomputing the trajectory.

The second difficulty is easily remedied. Since

$$G(v, a, \rho) = \rho v K_D(v/a) \tag{1.3}$$

we have

$$G(v, a, \rho_o) = \rho_o v K_D(v/a),$$

$$G(v, a, \rho_s) = \rho_s v K_D(v/a).$$

Therefore, we have

$$G(v, a, \rho_o) = \left[\rho_o/\rho_s\right] G(v, a, \rho_s). \tag{1.4}$$

Denoting $\rho_o/\rho_s$, the ratio of density at the muzzle to standard density at sea level, by $\sigma$, and replacing $G(v, a, \rho_o)$ in (1.2) by (1.4), we obtain

$$E = \frac{\rho(\eta)}{\rho_o} \sigma G(v, a, \rho_s) \frac{1}{c}. \tag{1.5}$$

It will presently be shown that writing $\Sigma$ as shown in (1.5) enables one to solve the ballistic equations once for quantities equivalent to $\sigma\xi$ and $\sigma\eta$ and then substitute for $\sigma$ to find the $\xi$ and $\eta$ appropriate to the altitude of the muzzle.

Now to relate $G(v, a, \rho_s)$ to the tabulated $G(v)$ we note that

(1.6) $$G(v) = G(v, a_s, \rho_s) = \rho_s\, v\, K_D(v/a_s).$$

Then replacing v in (1.6) by $[a_s/a]v$, we have

$$G([a_s/a]v) = \rho_s [a_s/a] v\, K_D(v/a).$$

But,

$$G(v, a, \rho_s) = \rho_s\, v\, K_D(v/a).$$

Therefore, we have

(1.7) $$G([a_s/a]v) = [a_s/a]\, G(v, a, \rho_s)\,.$$

Remembering that $a = a_s\sqrt{T/T_s}$ and denoting $\sqrt{T_s/T}$ by $\lambda$, we obtain from (1.7)

(1.8) $$\frac{G(\lambda v)}{\lambda} = G(v, a, \rho_s).$$

Finally, since the ratio of standard air density at any altitude R above sea level to standard sea level air density is given by

$$e^{-hR} \quad (h = 0.3158 \times 10^{-4} \text{ if R is given in feet}),$$

we can write

(1.9) $$\frac{\rho(\eta)}{\rho_0} = e^{h\eta}\,.$$

Substituting (1.8) and (1.9) in (1.5) we arrive at the following form for the ballistic equations

(1.10) $$\ddot{\xi} = -\frac{\sigma e^{h\eta}\, G(\lambda v)}{c\lambda}\,\dot{\xi}\,,$$

$$\ddot{\eta} = -\frac{\sigma e^{h\eta}\, G(\lambda v)}{c\lambda}\,\dot{\eta} + g.$$

If, instead of rectangular coordinates $(\xi, \eta)$ we use slant coordinates (x, y), the y-axis pointing vertically downward and the x-axis being taken along the tangent to the trajectory at the initial point, which tangent makes an angle $\varepsilon$ with the horizontal ($\varepsilon$ measured clockwise), then the equations for the transformation of variables are

$$\xi = x \cos \varepsilon \qquad \eta = \xi \tan \varepsilon + y$$

$$\dot{\xi} = \dot{x} \cos \varepsilon \qquad \dot{\eta} = \dot{\xi} \tan \varepsilon + \dot{y}$$

etc. etc.

and it can readily be shown that the equations (1.10) become

$$\ddot{x} = - \frac{\sigma e^{h\eta} G(\lambda v)}{c\lambda} \dot{x} ,$$
(1.11)
$$\ddot{y} = - \frac{\sigma e^{h\eta} G(\lambda v)}{c\lambda} \dot{y} + g.$$

The y-coordinate as used here is usually referred to as the drop.

Now, introduce a pseudo velocity u defined as the component of the velocity v along the tangent to the trajectory at the initial point. Clearly,

$$u = \dot{x},$$
(1.12)
$$\dot{u} = \ddot{x}.$$

Approximating* $e^{h\eta}$ by $e^{hx \sin \varepsilon}$ and approximating the actual velocity v by the psuedo velocity u as is done in the usual Siacci method (a satisfactory approximation if the trajectory is fairly flat and we restrict ourselves to short ranges), the equations of motion are approximately given by

$$\dot{u} = - \frac{\sigma e^{hx \sin\varepsilon} G(\lambda u)}{c\lambda} u,$$
(1.13)
$$\ddot{y} = - \frac{\sigma e^{hx \sin\varepsilon} G(\lambda u)}{c\lambda} \dot{y} + g.$$

* This approximation is cruder than that used by Hitchcock and Kent (see Ballistic Research Laboratory Report No. 114 ), but appears adequate, and is simpler computationally.

## 2. Constant Density Case

If it is assumed that air density along the trajectory does not vary from air density at the muzzle* (i.e. take $e^{hx \sin\theta} = 1$ or $h = 0$), the differential equations of motion may be integrated easily in terms of the Siacci Space and Time functions. These are defined

$$S(u) = -\int_{u_0}^{u} \frac{du}{G(u)}$$

$$T(u) = -\int_{u_0}^{u} \frac{du}{uG(u)} .$$

From (1.12) and (1.13) we obtain

$$\sigma\, dx = -\frac{c\,\lambda du}{G(\lambda u)},$$

and, integrating ,

(2.1) $$\sigma x = c\left[S(\lambda u) - S(\lambda u_0)\right].$$

This equation enables one to find $S(\lambda u)$ for a given slant range and initial velocity. Then $\lambda u$ may be found from Siacci Space function tables.

From (1.12) we have $dt = \frac{dx}{u}$ and, making use of (2.1), we find

(2.2) $$\sigma\, dt = \frac{c\, S'(\lambda u)\,\lambda du}{u}$$

which integrates into

(2.3) $$\sigma t = c\,\lambda\left[T(\lambda u) - T(\lambda u_0)\right].$$

Thus $\sigma t$ is found by entering the Siacci Time function table with the value of $\lambda u$ from equation (2.1).

* The treatment of a trajectory in which air density is not considered to be constant will be discussed in Section 4A.

Finally, using Dederick's identity*

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = g/u^2$$

we find

$$\sigma^2 y = g \int_0^x \int_0^x \frac{\sigma^2 dx\, dx}{u^2} \quad (2.4)$$

These equations are those used in making the fundamental table of $\sigma t$ and $\sigma^2 y$ against $\sigma x$, for a fixed initial velocity.

In Table 1 below, time of flight and drop corresponding to slant ranges up to 4500 yards have been computed in two ways for the 37 mm. Mark 9 HE 54 shell, fired horizontally ($\varepsilon = 0°$) at sea level with standard density ($\sigma = 1$) at temperature 59°F ($\lambda = 1$).

The 2nd and 3rd columns respectively show time of flight and drop computed by numerical integration of the normal equations of motion given in (1) and (2). Columns (4) and (5) show time of flight and drop computed from equations (1.13), (2.1),(2.3) and (2.4) which make use of the Siacci approximations mentioned in Section 1. Notice that through 3000 yards in slant range, drop and time of flight agree exactly for the two methods of computation. Even at a slant range of 4500 yards, the discrepancy in drop is under 1 %.

37 mm Mark 9 HE M54

Muzzle Velocity = 2550 fps, Airplane Velocity = 300 mph, $u_o$ = 2990 fps

$\varepsilon = 0°$ T = 59°F $\sigma = 1$ c = 1

| | Normal Equations with h = h | | Siacci with h = 0 | |
|---|---|---|---|---|
| $\sigma$ x yds. | $\sigma$ t sec. | $\sigma^2$ y yds. | $\sigma$ t sec. | $\sigma^2$ y yds. |
| 500 | 0.549 | 1.5 | 0.550 | 1.5 |
| 1000 | 1.208 | 6.9 | 1.208 | 7.0 |
| 1500 | 2.012 | 18.1 | 2.013 | 18.1 |
| 2000 | 3.010 | 37.8 | 3.010 | 37.9 |
| 2500 | 4.268 | 71.3 | 4.265 | 71.3 |
| 3000 | 5.777 | 125.6 | 5.769 | 125.4 |
| 3500 | 7.442 | 207.0 | 7.419 | 206.3 |
| 4000 | 9.244 | 320.8 | 9.203 | 318.9 |
| 4500 | 11.197 | 472.4 | 11.122 | 468.2 |

Table 1

* This identity may be arrived easily from equations (1.1) and (1.2) of Section 1.

3. Interpolation for Time of Flight and Drop.

In order for tabulation of $\sigma t$ and $\sigma^2 y$ against $\sigma x$ to be computationally successful it is necessary that this tabulation be done for as few velocities as possible. For many cases, as will be seen, tables need be constructed for only two velocities, provided interpolation is done in a special way.

A consideration of the trajectory in vacuo suggests a possible method of interpolation for time of flight and drop. The equations of motion for the trajectory in vacuo are:

$$\ddot{x} = 0 ,$$

$$\ddot{y} = g ,$$

and the initial conditions are:

$$x = 0 \qquad \dot{x} = u_0$$

$$y = 0 \qquad \dot{y} = 0 .$$

The solution of the equations is

$$t = \frac{x}{u_0} \tag{3.1}$$

$$y = 1/2\, gx^2 \left(\frac{1}{u_0}\right)^2 .$$

Equations (3.1) above lead one to suspect that, for a given slant range, it may be possible to compute time of flight by a linear interpolation on $1/u_0$ between two known times of flight corresponding to two initial velocities $u_{o_1}$ and $u_{o_2}$.

In Table 2 a harmonic linear interpolation based on times of flight corresponding to initial velocities of 2550 fps and 3430 fps was used to find time of flight for $\sigma x$ through 4500 yards.

The interpolation formula for finding $\sigma t$ corresponding to initial velocity $u_0$ is clearly

$$\sigma t = \left( 1 - \frac{\frac{1}{u_0} - \frac{1}{u_{o_1}}}{\frac{1}{u_{o_2}} - \frac{1}{u_{o_1}}} \right) \sigma t_1 + \left( \frac{\frac{1}{u_0} - \frac{1}{u_{o_1}}}{\frac{1}{u_{o_2}} - \frac{1}{u_{o_1}}} \right) \sigma t_2 \tag{3.2}$$

The results for this interpolation are shown in column 4 below. In column 3 are listed times of flight for an initial velocity of 2990 fps computed directly by means of equation (2.7). Notice that the results are the same to the nearest hundredth of a second through 2500 yard slant range. The discrepancy beyond 2500 yard range, however, may be due not to method but to a lack of smoothness in the neighborhood of sound in the G tables on which the Siacci computations for the 37 mm Mark 9 HE shell M54 were based.

TIME OF FLIGHT

By Siacci Formula and by Linear Interpolation on $(1/u_o)$

37 mm Mark 9 HE Shell M54, $c = 1$, $\varepsilon = 0^\circ$, $T = 59^\circ$, Constant Density

| $\sigma x$ yds. | for $u_{o_1} = 2550$ fps | for $u_o = 2990$ fps | | for $u_{o_2} = 3430$ fps |
|---|---|---|---|---|
| | SIACCI $\sigma t_1$ sec. | SIACCI $\sigma t$ sec. | LINEAR INTERPOLATION ON $(1/u_o)$ $\sigma t = \sigma t_1 + \Delta\sigma t$ sec. | SIACCI $\sigma t_2$ sec. |
| 500 | 0.646 | 0.550 | 0.549 | 0.477 |
| 1000 | 1.433 | 1.203 | 1.211 | 1.046 |
| 1500 | 2.409 | 2.013 | 2.021 | 1.732 |
| 2000 | 3.633 | 3.010 | 3.024 | 2.571 |
| 2500 | 5.116 | 4.265 | 4.256 | 3.616 |
| 3000 | 6.753 | 5.769 | 5.706 | 4.928 |
| 3500 | 8.524 | 7.419 | 7.345 | 6.469 |
| 4000 | 10.432 | 9.203 | 9.113 | 8.141 |

Table 2

In support of this view similar computations for the 75 mm HE shell M48 whose Siacci tables are smooth are offered in the footnote below.*

The approximation of drop by a quadratic function of $(1/u_o)$ is suggested by the solution for drop in the vacuum case given in (3.1). Such an approximation was used to find $\sigma^2 y$ by interpolation for the 37 mm shell.

The interpolation formula is

$$(3.3) \qquad \sigma^2 y = \left( 1 - \frac{\dfrac{1}{u_o^2} - \dfrac{1}{u_{o_1}^2}}{\dfrac{1}{u_{o_2}^2} - \dfrac{1}{u_{o_1}^2}} \right) \sigma^2 y_1 + \left( \frac{\dfrac{1}{u_o^2} - \dfrac{1}{u_{o_1}^2}}{\dfrac{1}{u_{o_2}^2} - \dfrac{1}{u_{o_1}^2}} \right) \sigma^2 y_2 \, .$$

The results of this interpolation are listed below in Table 3, column 7. Drop as computed by (2.4) is shown in column 4. The discrepancy between the Siacci computation and the interpolated drop to $\sigma x$ is shown in column 8. It is only at $\sigma x$ = 4000 yards that the discrepancy exceeds 1 % and here but slightly.

* TIME OF FLIGHT

75 mm HE Shell M48

c = 1.686  ε = 0°  T = 59°  Constant density

| $\sigma x$ yds. | for $u_{o_1}$ = 1950 fps SIACCI $\sigma t_1$ sec. | for $u_o$ = 2390 fps SIACCI $\sigma t$ sec. | for $u_o$ = 2390 fps HARMONIC LINEAR INT. $\sigma t$ sec. | for $u_o$ = 2830 fps SIACCI $\sigma t_2$ sec. |
|---|---|---|---|---|
| 1000 | 1.649 | 1.340 | 1.341 | 1.128 |
| 2000 | 3.552 | 2.871 | 2.876 | 2.411 |
| 3000 | 5.751 | 4.630 | 4.638 | 3.871 |
| 4000 | 8.287 | 6.660 | 6.664 | 5.545 |
| 5000 | 11.128 | 9.007 | 8.966 | 7.477 |

For ease of computation a harmonic linear interpolation for drop would be preferable to a harmonic quadratic for then the same coefficients could be used in interpolating for drop as are used in the time of flight interpolation. In column 5 below the results of harmonic linear interpolation are listed. The harmonic linear results are not quite as accurate as those of the harmonic quadratic interpolation except for $\sigma x \geqslant 3000$ yards where the linear interpolation appears to be more accurate. This reversal, however, appears to be an accident again due to lack of smoothness in the shell's Siacci tables.

Similar computations for the 75 mm shell show the harmonic quadratic interpolation to be distinctly more accurate at all ranges than the harmonic linear interpolation (Table 4).

The choice between a harmonic linear interpolation for drop and a harmonic quadratic interpolation depends upon the accuracy required in the results. In any case the use of harmonic interpolation on $u_o$ can produce a great saving in time and labor since it enables one to find time of flight and drop for intermediate airplane speeds simply from a tabulation of these quantities first for initial shell velocity with the airplane at rest and then with the airplane having another extreme velocity, say 600 mph (880 fps).

DROP

By Siacci Formulae and by Interpolation

37 mm Mark 9 HE Shell M54 $c = 1$, $\varepsilon = 0^\circ$, $T = 59^\circ$, Constant density

| | $u_{o_1}$ = 2550 fps | $u_{o_2}$ = 3430 fps | $u_o$ = 2990 fps | | | | |
|---|---|---|---|---|---|---|---|
| | | | SIACCI | HARMONIC LINEAR | DISCREPANCY | HARMONIC QUAD. | DISCREPANCY |
| $\sigma x$ yds. | $\sigma^2 y_1$ yds. | $\sigma^2 y_2$ yds. | $\sigma^2 y$ yds. | $\sigma^2 y$ yds. | % | $\sigma^2 y$ yds. | % |
| 500 | 2.11 | 1.15 | 1.53 | 1.56 | .1 | 1.53 | 0 |
| 1000 | 9.71 | 5.24 | 6.95 | 7.15 | .2 | 6.93 | 0 |
| 1500 | 25.60 | 13.54 | 18.11 | 18.68 | .4 | 18.25 | .1 |
| 2000 | 54.45 | 28.06 | 37.92 | 39.31 | .7 | 38.36 | .2 |
| 2500 | 103.27 | 52.06 | 71.31 | 73.90 | 1.0 | 72.05 | .3 |
| 3000 | 178.43 | 90.99 | 125.38 | 128.28 | 1.0 | 125.13 | .1 |
| 3500 | 284.72 | 152.11 | 206.32 | 208.66 | .7 | 203.88 | 1.2 |
| 4000 | 427.22 | 241.13 | 318.89 | 320.48 | .4 | 313.78 | 1.7 |

Table 3

DROP

By Siacci Formulae and by Interpolation

75 mm HE Shell M48 $c = 1.686$, $\varepsilon = 0^\circ$, $T = 59^\circ$ Constant density

| | $u_{o_1}$ = 1950 fps | $u_{o_2}$ = 2830 fps | $u_o$ = 2390 fps | | | | |
|---|---|---|---|---|---|---|---|
| | | | SIACCI | HARMONIC LINEAR | DISCREPANCY | HARMONIC QUAD. | DISCREPANCY |
| $\sigma x$ yds. | $\sigma^2 y_1$ yds. | $\sigma^2 y_2$ yds. | $\sigma^2 y$ yds. | $\sigma^2 y$ yds. | % | $\sigma^2 y$ yds. | % |
| 1000 | 13.9 | 6.6 | 9.2 | 9.6 | .4 | 9.3 | .1 |
| 2000 | 61.7 | 28.7 | 40.5 | 42.2 | .8 | 40.7 | .1 |
| 3000 | 154.5 | 70.9 | 100.7 | 105.0 | 1.4 | 101.3 | .2 |
| 4000 | 307.8 | 139.4 | 199.3 | 208.1 | 2.2 | 200.6 | .4 |
| 5000 | 542.6 | 242.7 | 349.3 | 365.0 | 3.1 | 351.7 | .5 |

Table 4

4. Differential Corrections.

A. Variable Density Along the Trajectory.

The equations of motion have so far been solved for the case in which air density at all altitudes was assumed to remain the same as standard density at the muzzle. This was accomplished by setting $h = 0$ in the equations of motion. Next we will attempt to consider the effect of variable air density.

Taking $\lambda = 1$ for simplicity (a correction for temperature will be discussed later), equation (1.13) leads to

$$(4.1) \qquad \int_0^x e^{hx} \sin \varepsilon \, dx = \frac{c}{\sigma}\left[S(u) - S(u_0)\right] .$$

For fixed values of initial velocity, $u_0$, and angle of elevation, $\varepsilon$, equation (4.1) determines $u$ as a function of $\sigma x$ and of $h$. Then $\sigma t$ and $\sigma^2 y$ given respectively by

$$(4.2) \qquad \sigma t = \int_0^x \frac{\sigma \, dx}{u(\sigma x, h)}$$

and

$$(4.3) \qquad \sigma^2 y = g \int_0^x \int_0^x \frac{\sigma^2 \, dx}{u^2(\sigma x, h)}$$

are also functions of $\sigma x$ and $h$.

At a given slant range $\sigma x$, then, the effect of a change in density on $\sigma t$ and $\sigma^2 y$ respectively, $\Delta_h \sigma t$ and $\Delta_h \sigma^2 y$, may be thought of as

$$\Delta_h \sigma t = \sigma t(\sigma x, h) - \sigma t(\sigma x, 0)$$

$$\Delta_h \sigma^2 y = \sigma^2 y(\sigma x, h) - \sigma^2 y(\sigma x, 0).$$

We approximate $\Delta_h \sigma t$ and $\Delta_h \sigma^2 y$ by the first order derivative terms in the Taylor expansions of $\sigma t(\sigma x, h)$ and $\sigma^2 y(\sigma x, h)$ respectively about $h = 0$.

Finding $\frac{\partial u}{\partial h}$ from (4.1) and using (4.2) we obtain

$$\sigma^2 \frac{\partial t}{\partial h}\bigg|_{h=0} = -\frac{\sin \varepsilon}{2c}\int_0^x \frac{\sigma^2 x^2 \, \sigma \, dx}{u^2(\sigma x, 0)\, S'(u)}$$

or, using $\sigma x = c\left[S(u) - S(u_0)\right]$ for $h = 0$

$$\sigma^2 \frac{\partial t}{\partial h}\bigg|_{h=0} = -c^2 \frac{\sin \varepsilon}{2}\int_{u_0}^{u} \left[\frac{S(u) - S(u_0)}{u(\sigma x, 0)}\right]^2 du.$$

This latter form has the advantage of eliminating $S'(u)$ from the computation.

From (4.3) we obtain

$$\sigma \frac{\partial \sigma^2 y}{\partial h}\bigg|_{h=0} = -\frac{g}{c}\sin \varepsilon \int_0^x \int_0^x \frac{\sigma^2 x^2 \, \sigma^2 dx \, dx}{u^3 \, S'(u)} .$$

Hence, the differential corrections for density on time of flight and drop respectively are given by

$$(4.4) \qquad \sigma \cdot \Delta_h \sigma t = -\frac{h \sin \varepsilon}{2c}\int_0^x \frac{\sigma^2 x^2 \, \sigma \, dx}{u^2 S'(u)}$$

and

$$(4.5) \qquad \sigma \cdot \Delta_h \sigma^2 y = -\frac{hg \sin \varepsilon}{c}\int_0^x \int_0^x \frac{\sigma^2 x^2 \, \sigma^2 \, dx \, dx}{u^3 S'(u)} .$$

In order to observe how well the first order approximation for the correction in time of flight works, we have tabulated below in Table 5, for the 37 mm Mark 9 HE M54 shell, time of flight computed from the normal differential equations, with density varying with altitude, compared with time of flight computed for constant density and corrected by (4.4). The initial angle was taken as 60° upward ($\varepsilon = -60°$), an extreme angle of fire and one for which extreme variation in density is to be expected. Through 2500 yards slant range there is exact agreement in time of flight. Beyond 2500 yards in slant range the differential correction seems very inadequate. The table also contains a comparison of drop. The y's in column 3 were computed from the normal equations with variable density. The y's in column 5 were computed with the Siacci equations with constant density and are not corrected for variable density. Through 2500 yards slant ranges it seems to be unnecessary to make the complicated

correction on drop since the discrepancy between actual drop/range and uncorrected drop/range is under 2%. Nor is there any point in correcting drop beyond 2500 yards since even the corrected time of flight beyond this range is quite inaccurate.

TIME OF FLIGHT AND DROP WITH VARIABLE AIR DENSITY

37 mm Mark 9 HE shell M54, c= 1, $\sigma$ = 1, $\varepsilon$ =-60°, airplane velocity=300 mph, muzzle velocity=2550 fps; $u_0$=2990 fps

| x yds. | NORMAL EQUATIONS t(1x,h) secs. | NORMAL EQUATIONS y(1x,h) yds. | SIACCI EQUATIONS $t(1x,h) = t(1x,0) + \Delta t$ | SIACCI EQUATIONS y(1x,0) yds. |
|---|---|---|---|---|
| 500 | 0.548 | 1.4 | 0.549 | 1.5 |
| 1000 | 1.201 | 6.7 | 1.201 | 7.0 |
| 1500 | 1.983 | 17.7 | 1.984 | 18.1 |
| 2000 | 2.920 | 36.5 | 2.923 | 37.9 |
| 2500 | 4.056 | 66.7 | 4.061 | 71.3 |
| 3000 | 5.383 | 113.5 | 5.449 | 125.4 |
| 3500 | 6.822 | 181.0 | 6.986 | 206.3 |
| 4000 | 8.335 | 271.8 | 8.616 | 318.9 |
| 4500 | 9.920 | 383.1 | 10.323 | 468.2 |

TABLE 5

B. Non-Standard Temperature.

In a somewhat analogous fashion to that of 4A we can arrive at an approximation for the effect of a deviation from standard temperature on time of flight and drop. This time, considering air density to be constant along the trajectory, we have, for a given range and a fixed initial velocity, from equations (2.1), (2.3), (2.4), $\sigma t$ and $\sigma^2 y$ defined as functions of slant range, $\sigma x$, and $\lambda$, the square root of the ratio of standard temperature to actual temperature.

The effect of a deviation in the ratio $\lambda = \sqrt{T_s/T}$ on time of flight and drop will be denoted by $\Delta_\lambda \sigma t$ and $\Delta_\lambda \sigma^2 y$ respectively and defined as follows:

$$(4.6) \qquad \Delta_\lambda \sigma t = \sigma t(\sigma x, \lambda) - \sigma t(\sigma x, 1)$$

and

$$(4.7) \qquad \Delta_\lambda \sigma^2 y = \sigma^2 y(\sigma x, \lambda) - \sigma^2 y(\sigma x, 1).$$

Again $\Delta_\lambda \sigma t$ and $\Delta_\lambda \sigma^2 y$ will be approximated by the first order terms in their respective Taylor expansions about $\lambda = 1$. Differentiation with respect to $\lambda$ in (2.1), (2.3), and (2.4) leads to

$$(4.8) \qquad \Delta_\lambda \sigma t = (\lambda - 1) \left\{ \sigma t(\sigma x, 1) + c\, S'(u_o) \left( \frac{u_o}{u} - 1 \right) \right\},$$

$$(4.9) \qquad \Delta_\lambda \sigma^2 y = 2(\lambda - 1) \left[ \sigma^2 y(\sigma x, 1) - g u_o S'(u_o) \int_o^x \int_o^x \frac{\sigma^2\, dx\, dx}{u^3 S'(u)} \right].$$

For the 37 mm Mark 9 HE M54, we have tabulated below in column 4 time of flight for $\lambda = 1.125$* as corrected by the first order approximation given in (4.8). Time of flight computed directly from formula (2.3) with $\lambda$ set equal to 1.125 appears in column 5. Through 2700 yards, the discrepancy between the entries in columns 4 and 5 is no more than one hundredth of a second. Beyond 2700 yards, the discrepancy becomes considerably larger.

* $\lambda = 1.125$ corresponds to a temperature of $-49.8^\circ$ F, as extreme a deviation from the standard temperature of $59^\circ$ F as is likely to be encountered.

TIME OF FLIGHT CORRECTED FOR NON-STANDARD TEMPERATURE

37 mm Mark 9 HE shell M54, $c = 1$, $\varepsilon = 0^{\circ}$, $u_o = 2990$ fps, constant density

| $\sigma x$ yds. | $\sigma t$ for $\lambda = 1$ | $\Delta_{\lambda}\sigma t$ for $\lambda = 1.125$ | $\sigma t(\sigma x, 1.125)$ $\sigma t(\sigma x,1) + \Delta_{\lambda}\sigma t$ | $\sigma t(\sigma x, 1.125)$ $c\lambda[T(\lambda u) - T(\lambda u_o)]$ |
|---|---|---|---|---|
| 1000 | 1.208 | -.010 | 1.198 | 1.193 |
| 1500 | 2.013 | -.207 | 1.986 | 1.991 |
| 2000 | 3.010 | -.064 | 2.946 | 2.958 |
| 2500 | 4.265 | -.108 | 4.157 | 4.166 |
| 2700 | 4.843 | -.102 | 4.741 | 4.731 |
| 2900 | 5.454 | -.075 | 5.379 | 5.348 |
| 3100 | 6.088 | -.036 | 6.052 | 6.014 |
| 3300 | 6.745 | +.009 | 6.754 | 6.708 |
| 3500 | 7.419 | +.056 | 7.475 | 7.432 |
| 3700 | 8.115 | .106 | 8.221 | 8.173 |
| 3900 | 8.834 | .157 | 8.991 | 8.942 |
| 4100 | 9.574 | .210 | 9.784 | 9.735 |
| 4300 | 10.339 | .265 | 10.604 | 10.549 |
| 4500 | 11.122 | .321 | 11.443 | 11.392 |

TABLE 6

Here again, a similar computation for the 75 mm shell suggests that roughness in the 37 mm Siacci Tables may be partial cause of the discrepancy beyond 2700 yards. Even as far out as $\sigma x = 5500$ yards, there is exact agreement to hundredths of a second between time of flight corrected for temperature and time of flight exactly computed for the actual temperature.

TIME OF FLIGHT CORRECTED FOR NON-STANDARD TEMPERATURE

75 mm HE Shell M48, $c = 1.686$, $\varepsilon = 0^{\circ}$, $u_o = 2390$ fps, constant density

| $\sigma x$ yds. | $\sigma t$ for $\lambda = 1$ secs. | $\sigma t$ for $\lambda = 1.125$ | |
|---|---|---|---|
| | | $\sigma t(\sigma x,1) + \Delta_{\lambda}\sigma t$ | $c\lambda[T(\lambda u) - T(\lambda u_o)]$ |
| 1000 | 1.340 | 1.337 | 1.336 |
| 1500 | 2.079 | 2.071 | 2.073 |
| 2000 | 2.871 | 2.856 | 2.859 |
| 2500 | 3.718 | 3.692 | 3.697 |
| 3000 | 4.630 | 4.589 | 4.596 |
| 3500 | 5.606 | 5.548 | 5.558 |
| 4000 | 6.660 | 6.584 | 6.591 |
| 4500 | 7.789 | 7.693 | 7.703 |
| 5000 | 9.007 | 8.894 | 8.896 |
| 5500 | 10.308 | 10.184 | 10.182 |
| 6000 | 11.696 | 11.607 | 11.580 |

TABLE 7

No correction on drop has been made since for $\sigma x$ less than 2700 yards, the drop correction is under 1 ₥.

## SUMMARY


It is suggested from the preceding study on the 37 mm projectile that the equations of motion be solved for $\sigma t$ and $\sigma^2 y$ as functions of $\sigma x$ first for constant air density along the trajectory, for standard air temperature and for standard muzzle velocity by the equations:

$$\sigma x = c\,[S(u) - S(u_0)]$$

$$\sigma t = c\,[T(u) - T(u_0)]$$

$$\sigma^2 y = g \int_0^x \int_0^x \frac{\sigma^2\, dx\, dx}{u^2} .$$

These results may be tabulated with initial velocity adjusted for two airplane speeds: 0 and 600 mph. Corrections may be added to these results for conditions different from the assumed ones.

The effect on time of flight of variable air density along the trajectory can be computed by the formula

$$\sigma \cdot \Delta_h \sigma t = - \frac{h \sin \varepsilon}{2c} \int_0^x \frac{(\sigma x)^2\, \sigma\, dx}{u^2 S'(u)} .$$

Even in the extreme case of 60° upward fire this correction yields accurate results through $\sigma x$ = 2500 yards. Density corrections on drop appear to be unnecessary. Even in the 60° upward angle of fire case, the discrepancy between corrected and uncorrected drop compared with $\sigma x$ is under 2 % as far out as 2500 yards.

The correction on time of flight for non-standard temperature is made by the formula

$$\Delta_\lambda \sigma t = (\lambda - 1) \left\{ \sigma t(\sigma x, 1) + cS'(u_0) \left(\frac{u_0}{u} - 1\right) \right\} .$$

The corrections so obtained will be good to approximately 2700 yards in $\sigma x$. The temperature correction on drop may very well be omitted since through $\sigma x$ = 2700 yards $\Delta_\lambda \sigma^2 y / \sigma x$ is under 1 %.

From the tabulations of $\sigma t$ and $\sigma^2 y$ for initial velocities obtained when the airspeed of the plane is 0 and 600 mph, one can find $\sigma t$ and $\sigma^2 y$ when the airplane has intermediate speeds.

The time of flight and drop are given by

$$\sigma t = \left(1 - \frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma t_1 + \left(\frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma t_2$$

and

$$\sigma^2 y = \left(1 - \frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma^2 y_1 + \left(\frac{1/u_o - 1/u_{o_1}}{1/u_{o_2} - 1/u_{o_1}}\right)\sigma^2 y_2$$

or, where greater accuracy is required,

$$\sigma^2 y = \left(1 - \frac{1/u_o^2 - 1/u_{o_1}^2}{1/u_{o_2}^2 - 1/u_{o_1}^2}\right)\sigma^2 y_1 + \left(\frac{1/u_o^2 - 1/u_{o_1}^2}{1/u_{o_2}^2 - 1/u_{o_1}^2}\right)\sigma^2 y_2 .$$

A. K. Goldstine

A. K. Goldstine

J. L. Kelley

J. L. Kelley

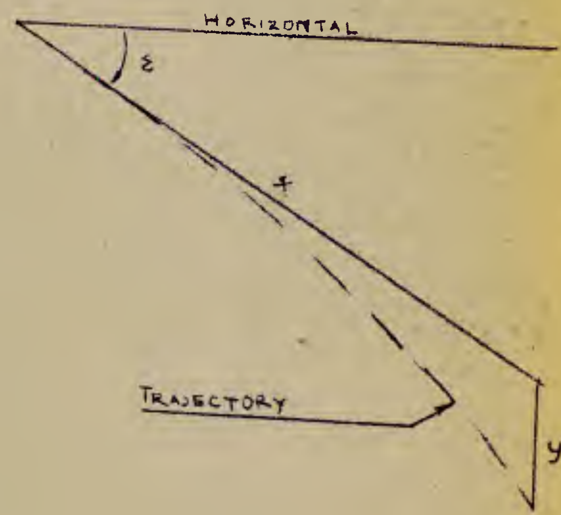
# APPENDIX

Ordnance Research Center
Project No. 4005

# BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED FORWARD FROM AN AIRPLANE

1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed $V_A$, at dive angle $\varepsilon$ .




$x$ = distance along line of fire

$y$ = drop from line of fire

$t_f = t_{f_o} + \Delta t_f \sin \varepsilon$ = time of flight

$t_{f_o}$ = time of flight for $\varepsilon = 0$

$\Delta t_f \sin \varepsilon$ = correction for variation of density

$\sigma$ = density ratio = density $\div$ standard density

2. Assumptions:

Temperature = 59°F.

Drift = 0

3. Accuracy:

At $(\sigma x) \leqq 5000$ ft., the error in $(\sigma^2 y)$ is $<1$ ft., the error in $(\sigma t_f) < .03$ secs. Beyond $\sigma x = 5000$ ft., accuracy falls off badly, and the table cannot be considered to represent the facts adequately.

Ordnance Research Center
Project No. 4005

# BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED FORWARD FROM AN AIRPLANE (Cont'd)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2870 f.p.s.

| $\sigma x$ ft. | $\sigma^2 y$ ft. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 100 | .0 | .027 | |
| 200 | .0 | .054 | |
| 300 | .1 | .082 | |
| 400 | .2 | .110 | |
| 500 | .3 | .138 | .000 |
| 600 | .4 | .167 | |
| 700 | .6 | .196 | |
| 800 | .8 | .225 | |
| 900 | 1.0 | .255 | |
| 1000 | 1.2 | .286 | .000 |
| 1100 | 1.5 | .316 | |
| 1200 | 1.8 | .348 | |
| 1300 | 2.2 | .380 | |
| 1400 | 2.6 | .412 | |
| 1500 | 3.0 | .444 | .001 |
| 1600 | 3.4 | .478 | |
| 1700 | 3.9 | .511 | |
| 1800 | 4.4 | .546 | |
| 1900 | 4.9 | .580 | |
| 2000 | 5.5 | .616 | .002 |
| 2100 | 6.2 | .652 | |
| 2200 | 6.9 | .688 | |
| 2300 | 7.6 | .725 | |
| 2400 | 8.3 | .763 | |
| 2500 | 9.1 | .801 | .005 |
| 2600 | 10.0 | .840 | |
| 2700 | 10.9 | .880 | |
| 2800 | 11.9 | .920 | |
| 2900 | 12.9 | .961 | |
| 3000 | 14.0 | 1.002 | .009 |
| 3100 | 15.1 | 1.045 | |
| 3200 | 16.3 | 1.088 | |
| 3300 | 17.5 | 1.132 | |
| 3400 | 18.8 | 1.176 | |
| 3500 | 20.2 | 1.222 | .015 |
| 3600 | 21.6 | 1.268 | |
| 3700 | 23.1 | 1.315 | |
| 3800 | 24.7 | 1.363 | |
| 3900 | 26.4 | 1.412 | |
| 4000 | 28.1 | 1.462 | .025 |

BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED FORWARD FROM AN AIRPLANE (Cont'd)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2870 f.p.s.

| $\sigma x$ ft. | $\sigma^2 y$ ft. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 4100 | 29.9 | 1.513 | |
| 4200 | 31.8 | 1.565 | |
| 4300 | 33.8 | 1.617 | |
| 4400 | 35.9 | 1.671 | |
| 4500 | 38.1 | 1.726 | .039 |
| 4600 | 40.4 | 1.782 | |
| 4700 | 42.7 | 1.838 | |
| 4800 | 45.2 | 1.896 | |
| 4900 | 47.8 | 1.956 | |
| 5000 | 50.5 | 2.016 | .060 |
| 5100 | 53.4 | 2.078 | |
| 5200 | 56.3 | 2.141 | |
| 5300 | 59.4 | 2.205 | |
| 5400 | 62.6 | 2.270 | |
| 5500 | 66.0 | 2.337 | .088 |
| 5600 | 69.5 | 2.406 | |
| 5700 | 73.1 | 2.475 | |
| 5800 | 77.0 | 2.546 | |
| 5900 | 80.9 | 2.619 | |
| 6000 | 85.1 | 2.693 | .128 |
| 6100 | 89.4 | 2.769 | |
| 6200 | 94.0 | 2.847 | |
| 6300 | 98.7 | 2.926 | |
| 6400 | 103.6 | 3.007 | |
| 6500 | 108.8 | 3.090 | .181 |
| 6600 | 114.2 | 3.175 | |
| 6700 | 119.7 | 3.261 | |
| 6800 | 125.6 | 3.348 | |
| 6900 | 131.7 | 3.439 | |
| 7000 | 138.1 | 3.530 | .243 |
| 7100 | 144.7 | 3.623 | |
| 7200 | 151.7 | 3.717 | |
| 7300 | 158.9 | 3.812 | |
| 7400 | 166.4 | 3.909 | |
| 7500 | 174.2 | 4.006 | .288 |

BALLISTIC DATA FOR CAL. 0.50 APl MARK 8 FIRED
FORWARD FROM AN AIRPLANE (Cont'd)

$V_A$ = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2870 f.p.s.

| $\sigma x$ ft. | $\sigma^2 y$ ft. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 100 | .0 | .030 | |
| 200 | .1 | .061 | |
| 300 | .1 | .093 | |
| 400 | .2 | .124 | |
| 500 | .4 | .157 | .000 |
| 600 | .6 | .190 | |
| 700 | .8 | .223 | |
| 800 | 1.0 | .256 | |
| 900 | 1.3 | .291 | |
| 1000 | 1.6 | .326 | .000 |
| 1100 | 2.0 | .361 | |
| 1200 | 2.4 | .397 | |
| 1300 | 2.8 | .434 | |
| 1400 | 3.3 | .471 | |
| 1500 | 3.8 | .508 | .001 |
| 1600 | 4.4 | .547 | |
| 1700 | 5.0 | .586 | |
| 1800 | 5.7 | .625 | |
| 1900 | 6.5 | .666 | |
| 2000 | 7.3 | .707 | .003 |
| 2100 | 8.1 | .748 | |
| 2200 | 9.0 | .791 | |
| 2300 | 9.9 | .834 | |
| 2400 | 11.0 | .878 | |
| 2500 | 12.0 | .923 | .006 |
| 2600 | 13.2 | .968 | |
| 2700 | 14.4 | 1.015 | |
| 2800 | 15.7 | 1.062 | |
| 2900 | 17.0 | 1.110 | |
| 3000 | 18.5 | 1.159 | .011 |
| 3100 | 20.0 | 1.209 | |
| 3200 | 21.6 | 1.260 | |
| 3300 | 23.3 | 1.312 | |
| 3400 | 25.0 | 1.364 | |
| 3500 | 26.8 | 1.418 | .019 |
| 3600 | 28.8 | 1.473 | |
| 3700 | 30.8 | 1.529 | |
| 3800 | 33.0 | 1.586 | |
| [illegible] | [illegible] | [illegible] | |
| 4000 | [illegible] | [illegible] | .032 |

BALLISTIC DATA FOR CAL. 0.50 API MARK 8 FIRED
FORWARD FROM AN AIRPLANE (Cont'd)

$V_A$ = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2870 f.p.s.

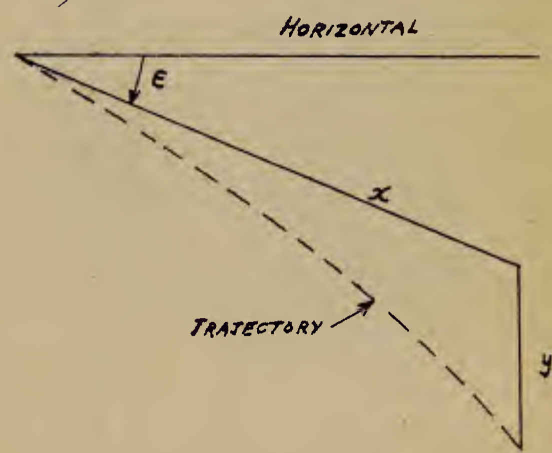
| $\sigma x$ ft. | $\sigma^2 y$ ft. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 4100 | 40.1 | 1.764 | |
| 4200 | 42.7 | 1.826 | |
| 4300 | 45.4 | 1.889 | |
| 4400 | 48.3 | 1.953 | |
| 4500 | 51.3 | 2.019 | .050 |
| 4600 | 54.4 | 2.086 | |
| 4700 | 57.7 | 2.154 | |
| 4800 | 61.2 | 2.224 | |
| 4900 | 64.8 | 2.295 | |
| 5000 | 68.5 | 2.368 | .076 |
| 5100 | 72.5 | 2.443 | |
| 5200 | 76.6 | 2.519 | |
| 5300 | 80.9 | 2.596 | |
| 5400 | 85.4 | 2.676 | |
| 5500 | 90.1 | 2.757 | .113 |
| 5600 | 95.1 | 2.840 | |
| 5700 | 100.2 | 2.924 | |
| 5800 | 105.6 | 3.011 | |
| 5900 | 111.3 | 3.099 | |
| 6000 | 117.2 | 3.189 | .159 |
| 6100 | 123.3 | 3.281 | |
| 6200 | 129.8 | 3.374 | |
| 6300 | 136.5 | 3.468 | |
| 6400 | 143.5 | 3.564 | |
| 6500 | 150.8 | 3.660 | .202 |
| 6600 | 158.4 | 3.757 | |
| 6700 | 166.3 | 3.856 | |
| 6800 | 174.5 | 3.955 | |
| 6900 | 183.1 | 4.053 | |
| 7000 | 192.0 | 4.155 | .226 |
| 7100 | 201.1 | 4.256 | |
| 7200 | 210.7 | 4.359 | |
| 7300 | 220.5 | 4.462 | |
| 7400 | 230.7 | 4.565 | |
| 7500 | 241.3 | 4.670 | .262 |

Ordnance Research and
Development Center
Project No. 4005

# BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4 FIRED FORWARD FROM AN AIRPLANE FROM 20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.)

1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed $V_A$, at dive angle $\varepsilon$.




$x$ = distance along line of fire.

$y$ = drop from line of fire.

$t_f = t_{f_o} + \Delta t_f \sin \varepsilon$ = time of flight.

$t_{f_o}$ = time of flight for $\varepsilon = 0$.

$\Delta t_f \sin \varepsilon$ = correction for variation of density.

$\sigma$ = density ratio = density at muzzle ÷ standard density at sea level.

2. Assumptions:

Temperature = 59° F.

Drift = 0.

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4,
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2750 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 50 | .0 | .042 | |
| 100 | .0 | .084 | |
| 150 | .1 | .128 | |
| 200 | .2 | .173 | .0001 |
| 250 | .3 | .219 | |
| 300 | .4 | .265 | |
| 350 | .5 | .313 | |
| 400 | .7 | .362 | .0005 |
| 450 | .9 | .412 | |
| 500 | 1.1 | .464 | |
| 550 | 1.3 | .517 | |
| 600 | 1.6 | .572 | .0018 |
| 650 | 1.9 | .628 | |
| 700 | 2.3 | .685 | |
| 750 | 2.7 | .744 | |
| 800 | 3.1 | .805 | .0048 |
| 850 | 3.5 | .867 | |
| 900 | 4.0 | .931 | |
| 950 | 4.6 | .997 | |
| 1000 | 5.2 | 1.066 | .0107 |
| 1050 | 5.8 | 1.136 | |
| 1100 | 6.5 | 1.208 | |
| 1150 | 7.3 | 1.282 | |
| 1200 | 8.1 | 1.359 | .0210 |
| 1250 | 9.0 | 1.439 | |
| 1300 | 9.9 | 1.521 | |
| 1350 | 10.9 | 1.605 | |
| 1400 | 12.1 | 1.692 | .0382 |
| 1450 | 13.3 | 1.782 | |
| 1500 | 14.6 | 1.876 | .0503 |
| 1550 | 16.0 | 1.973 | |
| 1600 | 17.4 | 2.072 | .0654 |
| 1650 | 19.0 | 2.175 | |
| 1700 | 20.7 | 2.282 | .0839 |
| 1750 | 22.6 | 2.393 | |
| 1800 | 24.6 | 2.507 | .1062 |
| 1850 | 26.7 | 2.625 | |
| 1900 | 28.9 | 2.747 | .1321 |
| 1950 | 31.4 | 2.873 | |
| [illegible] | 34.0 | 3.004 | .1597 |

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4,
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.) (Cont'd)

$V_A$ = Velocity of Airplane = 600 m.p.h.
Muzzle Velocity = 2750 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 2050 | 36.8 | 3.138 | |
| 2100 | 39.8 | 3.274 | .1852 |
| 2150 | 43.0 | 3.413 | |
| 2200 | 46.4 | 3.556 | .2086 |
| 2250 | 50.0 | 3.701 | |
| 2300 | 53.9 | 3.849 | .2317 |
| 2350 | 58.0 | 3.999 | |
| 2400 | 62.4 | 4.150 | .2554 |
| 2450 | 67.0 | 4.304 | |
| 2500 | 71.8 | 4.461 | .2806 |
| 2550 | 76.9 | 4.619 | |
| 2600 | 82.3 | 4.780 | .3083 |
| 2650 | 88.0 | 4.943 | |
| 2700 | 94.0 | 5.109 | .3385 |
| 2750 | 100.3 | 5.277 | |
| 2800 | 106.8 | 5.446 | .3716 |
| 2850 | 113.7 | 5.618 | |
| 2900 | 120.9 | 5.793 | .4082 |
| 2950 | 128.4 | 5.970 | |
| 3000 | 136.3 | 6.150 | .4491 |

BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.)

$V_A$ = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2750 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 50 | .0 | .047 | |
| 100 | .0 | .096 | |
| 150 | 0.1 | .146 | |
| 200 | 0.2 | .197 | .0001 |
| 250 | 0.3 | .250 | |
| 300 | 0.5 | .304 | |
| 350 | 0.7 | .359 | |
| 400 | 0.9 | .416 | .0006 |
| 450 | 1.1 | .475 | |
| 500 | 1.4 | .535 | |
| 550 | 1.7 | .597 | |
| 600 | 2.1 | .661 | .0023 |
| 650 | 2.5 | .727 | |
| 700 | 3.0 | .794 | |
| 750 | 3.5 | .864 | |
| 800 | 4.1 | .935 | .0061 |
| 850 | 4.7 | 1.009 | |
| 900 | 5.4 | 1.085 | |
| 950 | 6.1 | 1.164 | |
| 1000 | 6.9 | 1.245 | .0136 |
| 1050 | 7.8 | 1.329 | |
| 1100 | 8.8 | 1.415 | |
| 1150 | 9.9 | 1.504 | |
| 1200 | 11.0 | 1.596 | .0268 |
| 1250 | 12.2 | 1.692 | |
| 1300 | 13.6 | 1.791 | |
| 1350 | 15.0 | 1.893 | |
| 1400 | 16.6 | 1.999 | .0488 |
| 1450 | 18.3 | 2.109 | |
| 1500 | 20.1 | 2.222 | .0640 |
| 1550 | 22.0 | 2.339 | |
| 1600 | 24.1 | 2.460 | .0821 |
| 1650 | 26.4 | 2.585 | |
| 1700 | 28.8 | 2.714 | .1021 |
| 1750 | 31.4 | 2.847 | |
| 1800 | 34.3 | 2.983 | .1214 |
| 1850 | 37.3 | 3.122 | |
| 1900 | 40.6 | 3.263 | .1391 |
| 1950 | 44.1 | 3.407 | |
| 2000 | 47.7 | 3.554 | .1565 |

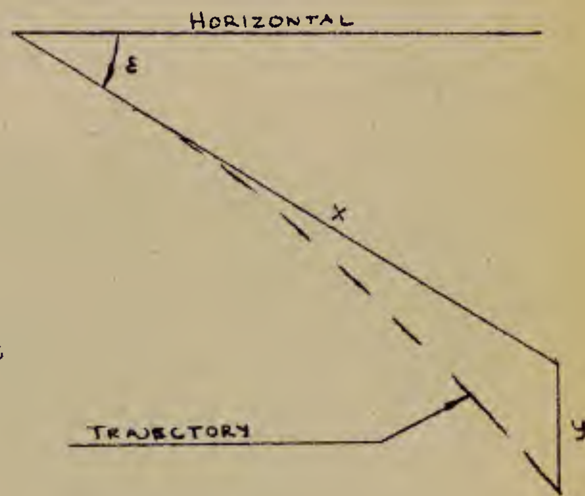
BALLISTIC DATA FOR 20 MM. H.E. SHELL T23, P.D. FUZE T71E4
FIRED FORWARD FROM AN AIRPLANE FROM
20 MM. AIRCRAFT GUN (TWIST OF RIFLING = 1 TURN IN 25.586 CAL.) (Cont'd)

$V_A$ = Velocity of Airplane = 300 m.p.h.
Muzzle Velocity = 2750 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 2050 | 51.6 | 3.703 | |
| 2100 | 55.3 | 3.854 | .1747 |
| 2150 | 60.2 | 4.008 | |
| 2200 | 64.9 | 4.165 | .1942 |
| 2250 | 69.8 | 4.323 | |
| 2300 | 75.0 | 4.482 | .2156 |
| 2350 | 80.5 | 4.644 | |
| 2400 | 86.2 | 4.809 | .2393 |
| 2450 | 92.2 | 4.976 | |
| 2500 | 98.6 | 5.145 | .2654 |
| 2550 | 105.3 | 5.317 | |
| 2600 | 112.3 | 5.491 | .2948 |
| 2650 | 119.6 | 5.667 | |
| 2700 | 127.2 | 5.846 | .3276 |
| 2750 | 135.2 | 6.027 | |
| 2800 | 143.6 | 6.211 | .3639 |
| 2850 | 152.3 | 6.398 | |
| 2900 | 161.4 | 6.587 | .4046 |
| 2950 | 170.9 | 6.779 | |
| 3000 | 180.8 | 6.974 | .4502 |

Ordnance Research Center
Project No. 4005

BALLISTIC DATA FOR 37MM H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37MM GUN MK. 9




1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed $V_A$, at dive angle $\varepsilon$.

$x$ = distance along line of fire

$y$ = drop from line of fire

$t_f = t_{f_o} + \Delta t_f \sin \varepsilon$ = time of flight

$t_{f_o}$ = time of flight for $\varepsilon = 0$

$\Delta t_f \sin \varepsilon$ = correction for variation of density

$\sigma$ = density ratio = density ÷ standard density

2. Assumptions:

Temperature = 59°F.

Drift = 0

3. Accuracy:

At $(\sigma x) \leqq 2000$ yds., the error in $(\sigma^2 y)$ is $< .5$ yd., the error in $(\sigma t_f) < .02$ secs. Beyond $\sigma x = 2000$ yds., accuracy falls off badly, and the table cannot be considered to represent the facts adequately.

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN, MK. 9 (CONT'D)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2550 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 100 | .04 | .089 | |
| 200 | .17 | .181 | |
| 300 | .39 | .276 | |
| 400 | .72 | .374 | |
| 500 | 1.15 | .477 | .0007 |
| 600 | 1.70 | .583 | |
| 700 | 2.37 | .692 | |
| 800 | 3.18 | .806 | |
| 900 | 4.13 | .924 | |
| 1000 | 5.24 | 1.046 | .0066 |
| 1100 | 6.51 | 1.174 | |
| 1200 | 7.96 | 1.305 | |
| 1300 | 9.60 | 1.441 | |
| 1400 | 11.45 | 1.584 | |
| 1500 | 13.54 | 1.732 | .0268 |
| 1600 | 15.86 | 1.887 | |
| 1700 | 18.45 | 2.047 | |
| 1800 | 21.34 | 2.215 | |
| 1900 | 24.52 | 2.389 | |
| 2000 | 28.06 | 2.571 | .0789 |
| 2100 | 31.96 | 2.762 | |
| 2200 | 36.27 | 2.961 | |
| 2300 | 41.04 | 3.170 | |
| 2400 | 46.28 | 3.389 | |
| 2500 | 52.06 | 3.616 | .1957 |
| 2550 | 55.16 | 3.737 | |
| 2600 | 58.43 | 3.856 | |
| 2650 | 61.85 | 3.981 | |
| 2700 | 65.43 | 4.108 | .2697 |
| 2750 | 69.20 | 4.236 | |
| 2800 | 73.16 | 4.371 | |
| 2850 | 77.31 | 4.506 | |
| 2900 | 81.66 | 4.645 | .3543 |

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN MK. 9 (CONT'D)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 2550 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 2950 | 86.22 | 4.786 | |
| 3000 | 90.99 | 4.928 | |
| 3050 | 96.00 | 5.073 | |
| 3100 | 101.22 | 5.223 | .4235 |
| 3150 | 106.70 | 5.373 | |
| 3200 | 112.41 | 5.525 | |
| 3250 | 118.38 | 5.679 | |
| 3300 | 124.59 | 5.833 | .4878 |
| 3350 | 131.07 | 5.991 | |
| 3400 | 137.81 | 6.148 | |
| 3450 | 144.83 | 6.308 | |
| 3500 | 152.11 | 6.469 | .5538 |
| 3550 | 159.68 | 6.631 | |
| 3600 | 167.53 | 6.795 | |
| 3650 | 175.67 | 6.959 | |
| 3700 | 184.11 | 7.126 | .6254 |
| 3750 | 192.82 | 7.294 | |
| 3800 | 201.88 | 7.464 | |
| 3850 | 211.22 | 7.633 | |
| 3900 | 220.86 | 7.801 | .6987 |
| 3950 | 230.83 | 7.970 | |
| 4000 | 241.12 | 8.141 | |
| 4050 | 251.74 | 8.315 | |
| 4100 | 262.59 | 8.494 | .7800 |
| 4150 | 273.97 | 8.676 | |
| 4200 | 285.60 | 8.855 | |
| 4250 | 297.56 | 9.037 | |
| 4300 | 309.88 | 9.219 | .8704 |
| 4350 | 322.57 | 9.401 | |
| 4400 | 335.60 | 9.584 | |
| 4450 | 349.01 | 9.772 | |
| 4500 | 362.79 | 9.960 | .9703 |

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN MK. 9 (CONT'D)

$V_A$ = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2550 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 100 | .06 | .102 | |
| 200 | .27 | .208 | |
| 300 | .52 | .318 | |
| 400 | .95 | .432 | |
| 500 | 1.53 | .550 | .0008 |
| 600 | 2.25 | .672 | |
| 700 | 3.15 | .799 | |
| 800 | 4.21 | .930 | |
| 900 | 5.48 | 1.066 | |
| 1000 | 6.95 | 1.208 | .0080 |
| 1100 | 8.65 | 1.357 | |
| 1200 | 10.56 | 1.511 | |
| 1300 | 12.08 | 1.671 | |
| 1400 | 15.29 | 1.838 | |
| 1500 | 18.11 | 2.013 | .0332 |
| 1600 | 21.26 | 2.195 | |
| 1700 | 24.77 | 2.386 | |
| 1800 | 28.70 | 2.585 | |
| 1900 | 33.07 | 2.793 | |
| 2000 | 37.92 | 3.010 | .1000 |
| 2100 | 43.31 | 3.238 | |
| 2200 | 49.28 | 3.478 | |
| 2300 | 55.90 | 3.728 | |
| 2400 | 63.22 | 3.990 | |
| 2500 | 71.31 | 4.265 | .2356 |
| 2550 | 75.66 | 4.406 | |
| 2600 | 80.22 | 4.549 | |
| 2650 | 85.02 | 4.695 | |
| 2700 | 90.05 | 4.843 | .2937 |
| 2750 | 95.30 | 4.993 | |
| 2800 | 100.81 | 5.145 | |
| 2850 | 106.57 | 5.299 | |
| 2900 | 112.58 | 5.454 | .3448 |

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM. GUN, MK. 9 (CONT'D)

$V_A$ = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 2550 f.p.s.

| $\sigma x$ Yds. | $\sigma^2 y$ Yds. | $\sigma t_{f_o}$ Sec. | $\sigma^2 \Delta t_f$ Sec. |
|---|---|---|---|
| 2950 | 118.84 | 5.611 | |
| 3000 | 125.38 | 5.769 | |
| 3050 | 132.18 | 5.925 | |
| 3100 | 139.26 | 6.088 | .3931 |
| 3150 | 146.62 | 6.250 | |
| 3200 | 154.26 | 6.413 | |
| 3250 | 162.19 | 6.578 | |
| 3300 | 170.42 | 6.745 | .4435 |
| 3350 | 178.93 | 6.910 | |
| 3400 | 187.76 | 7.077 | |
| 3450 | 196.88 | 7.247 | |
| 3500 | 206.32 | 7.419 | .5005 |
| 3550 | 216.07 | 7.592 | |
| 3600 | 226.15 | 7.765 | |
| 3650 | 236.55 | 7.939 | |
| 3700 | 247.28 | 8.115 | .5647 |
| 3750 | 258.35 | 8.293 | |
| 3800 | 269.75 | 8.472 | |
| 3850 | 281.50 | 8.652 | |
| 3900 | 293.61 | 8.834 | .6375 |
| 3950 | 306.06 | 9.018 | |
| 4000 | 318.89 | 9.203 | |
| 4050 | 332.07 | 9.388 | |
| 4100 | 345.64 | 9.574 | .7203 |
| 4150 | 359.57 | 9.762 | |
| 4200 | 373.90 | 9.953 | |
| 4250 | 388.60 | 10.146 | |
| 4300 | 403.72 | 10.339 | .3148 |
| 4350 | 419.22 | 10.532 | |
| 4400 | 435.14 | 10.726 | |
| 4450 | 451.47 | 10.922 | |
| 4500 | 468.22 | 11.122 | .9227 |

BALLISTIC DATA FOR 37 MM. H.E. SHELL, M54, FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 37 MM GUN, MK. 9 (CONT'D)

## Temperature Effect on Time of Flight

$t_f$ (at Temp. T) $= t_{f_o} + \sin\varepsilon\, \Delta t_f + (\lambda - 1)\, \delta t_f$

$\lambda = \sqrt{\dfrac{\text{Standard Temp., Abs.}}{\text{Temp., Abs.}}}$

Standard Temperature $= 59^\circ$ F.

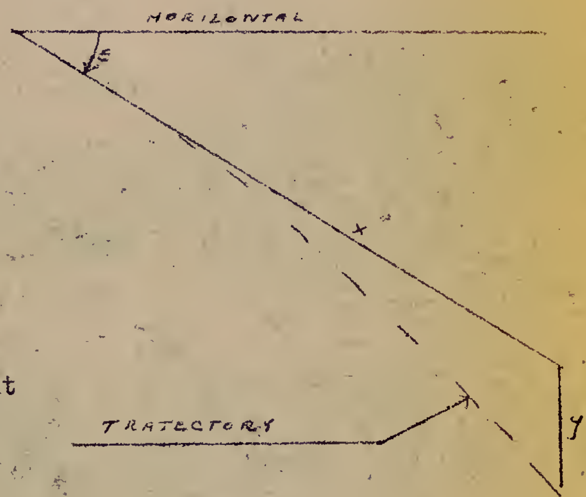
[Thus the $\lambda$ corresponding to $0^\circ$ F. is $\sqrt{\dfrac{518.4}{459.4}} = 1.064$]

$\sigma\, \delta t_f$
(Sec.)

| $\sigma x$ (Yds.) \ $V_A$ (m.p.h.) | 300 | 600 |
|---|---|---|
| 500 | -.01 | .00 |
| 1000 | -.08 | -.02 |
| 1500 | -.22 | -.11 |
| 2000 | -.51 | -.28 |
| 2500 | -.87 | -.62 |
| 2700 | -.82 | -.78 |
| 2900 | -.60 | -.90 |
| 3100 | -.29 | -.85 |
| 3300 | .07 | -.63 |
| 3500 | .45 | -.32 |
| 3700 | .85 | .03 |
| 3900 | 1.26 | .42 |
| 4100 | 1.68 | .80 |
| 4300 | 2.12 | 1.22 |
| 4500 | 2.57 | 1.64 |

Ordnance Research Center
Project No. 4005

# BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1.




## 1. Definitions:

The gun is fired along the line of flight of an airplane flying with speed $V_A$, at dive angle $\epsilon$.

$x$ = distance along line of fire

$y$ = drop from line of fire

$t_f = t_{f_o} + \Delta t_f \sin \epsilon$ = time of flight

$t_{f_o}$ = time of flight for $\epsilon = 0$

$\Delta t_f \sin \epsilon$ = correction for variation of density

$\sigma$ = density ratio = density at muzzle ÷ standard density at sea level

## 2. Assumptions:

Temperature = 59°F.

Drift = 0

## 3. Accuracy:

Through $\sigma x = 4500$ yds., $\sigma t_{f_o}$ is correct to 0.02 secs. and $\sigma^2 y$ to 1 yd. Beyond $\sigma x = 4500$ yds., the accuracy falls off so that by $\sigma x = 6000$ yds., $\sigma t_{f_o}$ is in error by about 0.07 secs. and $\sigma^2 y$ by 3.5 yds.

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1, (CONT'D)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 1930 f.p.s.

| $\sigma x$ yds. | $\sigma^2 y$ yds. | $\sigma t_{f_0}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 100 | 0.1 | 0.106 | |
| 200 | 0.2 | 0.214 | |
| 300 | 0.6 | 0.324 | |
| 400 | 1.0 | 0.435 | |
| 500 | 1.6 | 0.548 | .0003 |
| 600 | 2.3 | 0.661 | |
| 700 | 3.1 | 0.776 | |
| 800 | 4.1 | 0.892 | |
| 900 | 5.3 | 1.010 | |
| 1000 | 6.6 | 1.128 | .0023 |
| 1100 | 8.0 | 1.249 | |
| 1200 | 9.6 | 1.371 | |
| 1300 | 11.4 | 1.495 | |
| 1400 | 13.3 | 1.622 | |
| 1500 | 15.4 | 1.748 | .0083 |
| 1600 | 17.7 | 1.878 | |
| 1700 | 20.2 | 2.008 | |
| 1800 | 22.8 | 2.140 | |
| 1900 | 25.6 | 2.274 | |
| 2000 | 28.7 | 2.411 | .0208 |
| 2100 | 31.9 | 2.548 | |
| 2200 | 35.3 | 2.687 | |
| 2300 | 39.0 | 2.827 | |
| 2400 | 42.8 | 2.971 | |
| 2500 | 46.9 | 3.117 | .0439 |
| 2600 | 51.2 | 3.262 | |
| 2700 | 55.8 | 3.412 | |
| 2800 | 60.6 | 3.563 | |
| 2900 | 65.6 | 3.716 | |
| 3000 | 70.9 | 3.871 | .0806 |

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 1950 f.p.s.

| $\sigma x$ yds. | $\sigma^2 y$ yds. | $\sigma t_{f_0}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 3100 | 76.4 | 4.028 | |
| 3200 | 82.3 | 4.186 | |
| 3300 | 88.4 | 4.348 | |
| 3400 | 94.7 | 4.513 | |
| 3500 | 101.4 | 4.680 | .1364 |
| 3600 | 108.4 | 4.847 | |
| 3700 | 115.6 | 5.018 | |
| 3800 | 123.2 | 5.191 | |
| 3900 | 131.2 | 5.367 | |
| 4000 | 139.4 | 5.545 | .2189 |
| 4100 | 148.0 | 5.726 | |
| 4200 | 157.0 | 5.908 | |
| 4300 | 166.3 | 6.097 | |
| 4400 | 176.0 | 6.285 | |
| 4500 | 186.1 | 6.476 | .3344 |
| 4600 | 196.6 | 6.672 | |
| 4700 | 207.5 | 6.867 | |
| 4800 | 218.8 | 7.066 | |
| 4900 | 230.5 | 7.270 | |
| 5000 | 242.7 | 7.477 | .4905 |
| 5050 | 249.0 | 7.582 | |
| 5100 | 255.4 | 7.686 | |
| 5150 | 261.9 | 7.791 | |
| 5200 | 268.5 | 7.897 | .5661 |
| 5250 | 275.3 | 8.005 | |
| 5300 | 282.2 | 8.113 | |
| 5350 | 289.2 | 8.221 | |
| 5400 | 296.3 | 8.331 | .6502 |
| 5450 | 303.6 | 8.442 | |
| 5500 | 310.9 | 8.553 | |

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

$V_A$ = Velocity of Airplane = 600 m.p.h.

Muzzle Velocity = 1950 f.p.s.

| $\sigma x$ yds. | $\sigma^2 y$ yds. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 5550 | 318.5 | 8.666 | |
| 5600 | 326.1 | 8.779 | .7429 |
| 5650 | 333.9 | 8.892 | |
| 5700 | 341.9 | 9.005 | |
| 5750 | 349.9 | 9.120 | |
| 5800 | 358.2 | 9.182 | .8450 |
| 5850 | 366.5 | 9.352 | |
| 5900 | 375.0 | 9.470 | |
| 5950 | 383.7 | 9.590 | |
| 6000 | 392.5 | 9.710 | .9571 |

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56,
FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

$V_A$ = Velocity of Airplane = 300 m.p.h.
Muzzle Velocity = 1950 f.p.s.

| $\sigma x$ yds. | $\sigma^2 y$ yds. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 100 | 0.1 | 0.126 | |
| 200 | 0.3 | 0.253 | |
| 300 | 0.8 | 0.384 | |
| 400 | 1.4 | 0.516 | |
| 500 | 2.2 | 0.647 | .0005 |
| 600 | 3.2 | 0.784 | |
| 700 | 4.4 | 0.919 | |
| 800 | 5.8 | 1.057 | |
| 900 | 7.4 | 1.199 | |
| 1000 | 9.2 | 1.340 | .0031 |
| 1100 | 11.2 | 1.484 | |
| 1200 | 13.5 | 1.630 | |
| 1300 | 16.0 | 1.777 | |
| 1400 | 18.7 | 1.929 | |
| 1500 | 21.7 | 2.079 | .0112 |
| 1600 | 24.9 | 2.234 | |
| 1700 | 28.4 | 2.391 | |
| 1800 | 32.2 | 2.548 | |
| 1900 | 36.2 | 2.708 | |
| 2000 | 40.5 | 2.871 | .0272 |
| 2100 | 45.1 | 3.036 | |
| 2200 | 49.9 | 3.203 | |
| 2300 | 55.1 | 3.372 | |
| 2400 | 60.6 | 3.544 | |
| 2500 | 66.4 | 3.718 | .0559 |
| 2600 | 72.6 | 3.895 | |
| 2700 | 79.1 | 4.075 | |
| 2800 | 85.9 | 4.255 | |
| 2900 | 93.1 | 4.441 | |
| 3000 | 100.7 | 4.630 | .1033 |

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, P.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

$V_A$ = Velocity of Airplane = 300 m.p.h.
Muzzle Velocity = 1950 f.p.s.

| $\sigma$ x yds. | $\sigma$ 2y yds. | $\sigma$ $t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 3100 | 108.6 | 4.819 | |
| 3200 | 116.9 | 5.012 | |
| 3300 | 125.7 | 5.208 | |
| 3400 | 134.8 | 5.404 | |
| 3500 | 144.4 | 5.606 | .1747 |
| 3600 | 154.5 | 5.812 | |
| 3700 | 165.0 | 6.019 | |
| 3800 | 175.9 | 6.228 | |
| 3900 | 187.4 | 6.442 | |
| 4000 | 199.3 | 6.660 | .2776 |
| 4100 | 211.7 | 6.879 | |
| 4200 | 224.7 | 7.103 | |
| 4300 | 238.2 | 7.329 | |
| 4400 | 252.2 | 7.557 | |
| 4500 | 266.9 | 7.789 | .4173 |
| 4600 | 282.1 | 8.027 | |
| 4700 | 298.0 | 8.265 | |
| 4800 | 314.4 | 8.509 | |
| 4900 | 331.5 | 8.755 | |
| 5000 | 349.3 | 9.007 | .6021 |
| 5050 | 358.4 | 9.133 | |
| 5100 | 367.7 | 9.260 | |
| 5150 | 377.2 | 9.388 | |
| 5200 | 386.8 | 9.516 | .6897 |
| 5250 | 396.7 | 9.646 | |
| 5300 | 406.7 | 9.777 | |
| 5350 | 416.9 | 9.909 | |
| 5400 | 427.3 | 10.042 | .7839 |
| 5450 | 437.9 | 10.175 | |
| 5500 | 448.7 | 10.308 | |

BALLISTIC DATA FOR 75MM H.E. SHELL, M48, F.D. FUZE M56, FIRED FORWARD FROM AN AIRPLANE FROM THE 75MM GUN M5A1. (CONT'D)

$V_A$ = Velocity of Airplane = 300 m.p.h.

Muzzle Velocity = 1950 f.p.s.

| $\sigma x$ yds. | $\sigma^2 y$ yds. | $\sigma t_{f_o}$ sec. | $\sigma^2 \Delta t_f$ sec. |
|---|---|---|---|
| 5550 | 459.6 | 10.445 | |
| 5600 | 470.8 | 10.581 | .8853 |
| 5650 | 482.2 | 10.718 | |
| 5700 | 493.7 | 10.854 | |
| 5750 | 505.5 | 10.994 | |
| 5800 | 517.5 | 11.134 | .9820 |
| 5850 | 529.7 | 11.274 | |
| 5900 | 542.1 | 11.414 | |
| 5950 | 554.7 | 11.554 | |
| 6000 | 567.5 | 11.696 | 1.0571 |